

# An Active Learning Approach to the Falsification of Black Box Cyber-Physical Systems

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# Outline

## 1 Overview

- Model Based Development
- Signal Temporal Logic
- Search-Based Testing

## 2 Domain Estimation Problem

- Algorithm Idea

## 3 Test Case & Results

## 4 Challenges & Further studies

# Overview

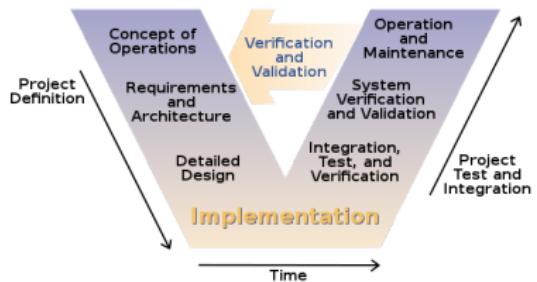
## Model Based Development

Methodology based on a computational model of a real target system

- used at the early stage of the design phase
- used at the end to verify the compliance of the real system

## Motivations

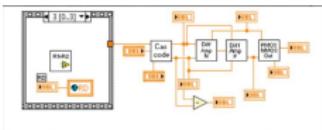
- reducing the time of prototyping
- reducing the cost of development



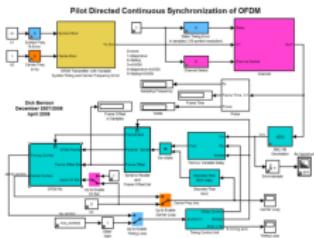
# Models

## Software: Block Diagram Systems

### LabView



### Simulink



## Computational Models

- Hybrid Systems
- CPS
- Automata
- Statistical Models

## Problem

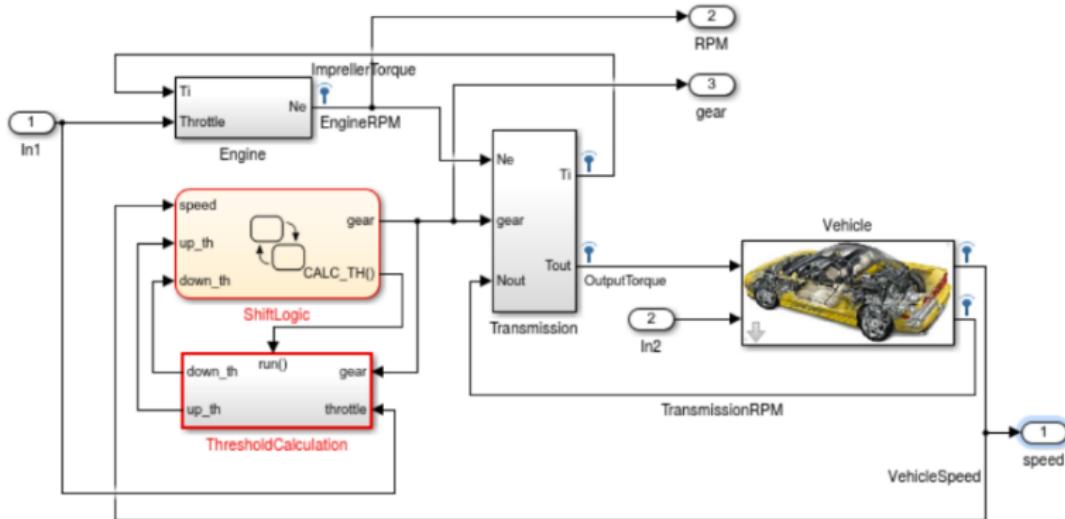
Too Much Complexity  $\Rightarrow$  no standard Model checking techniques.



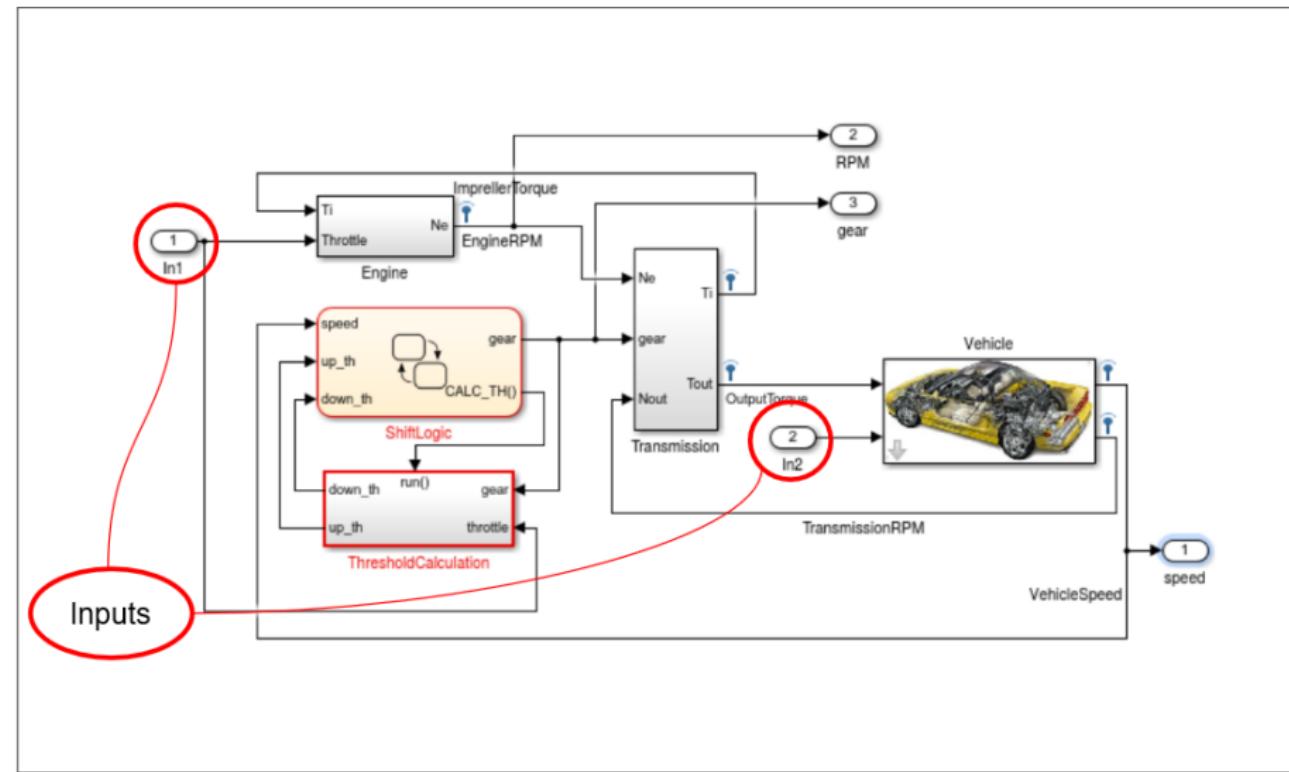
## Solution

**Black Box Assumption and Search-based approach.**

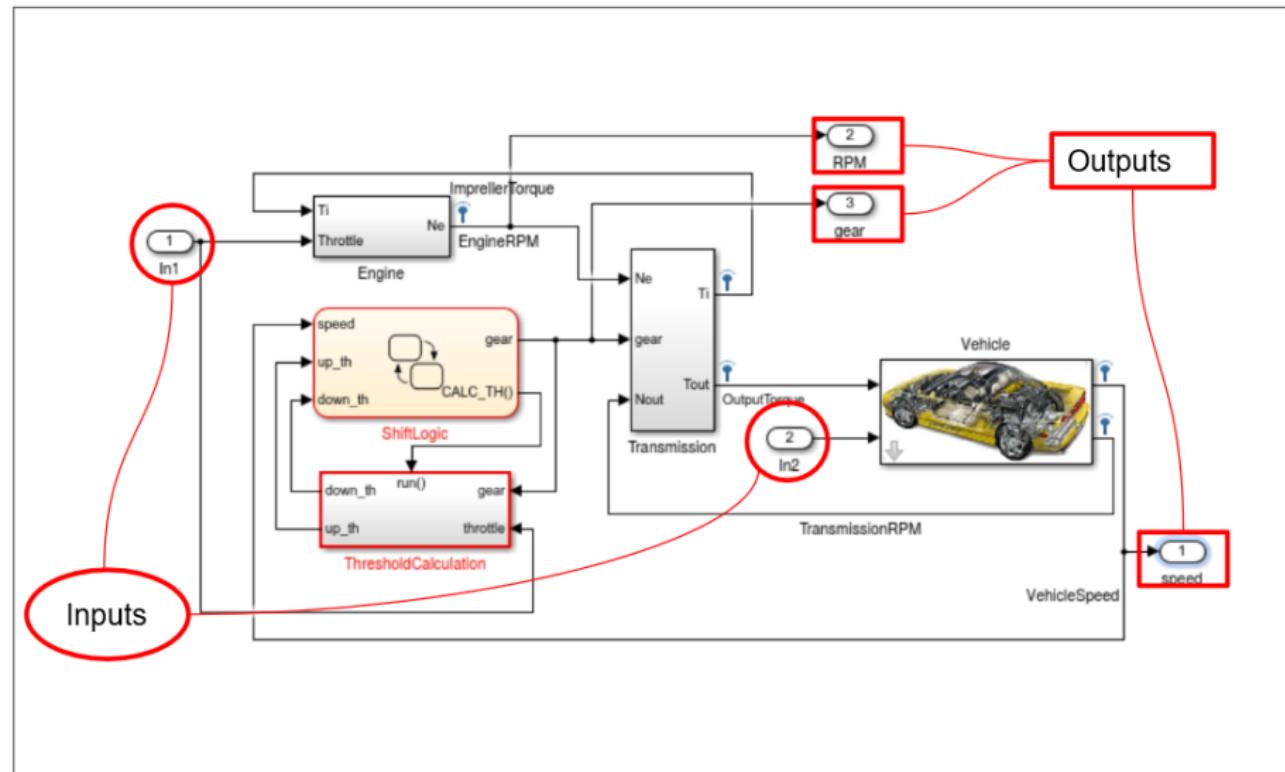
# Simulink Model



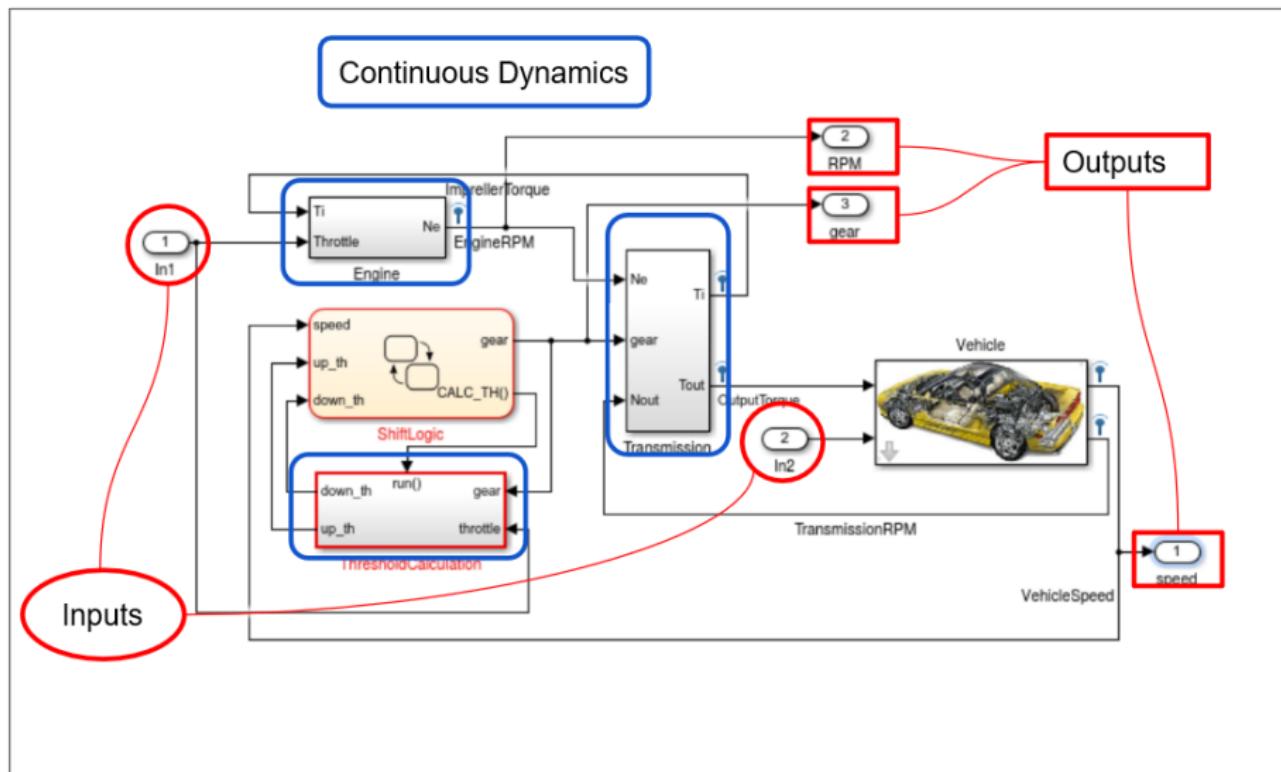
# Simulink Model - Inputs



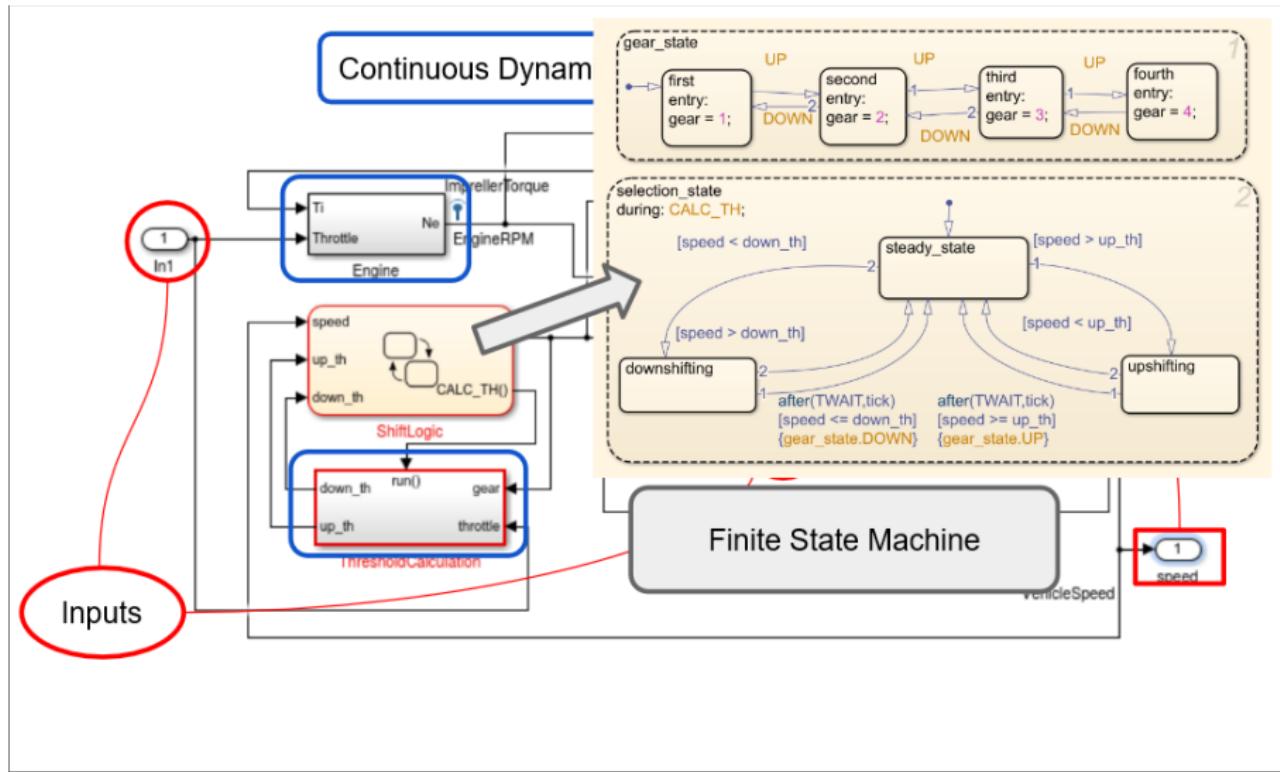
# Simulink Model - Outputs



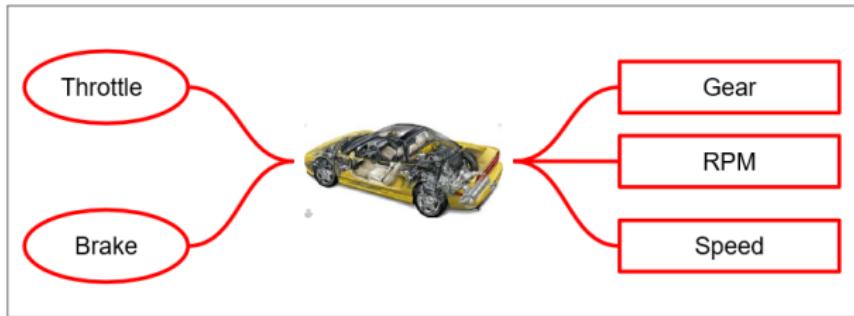
# Simulink Model - Continuous Dynamics



# Simulink Model - Finite State Machine



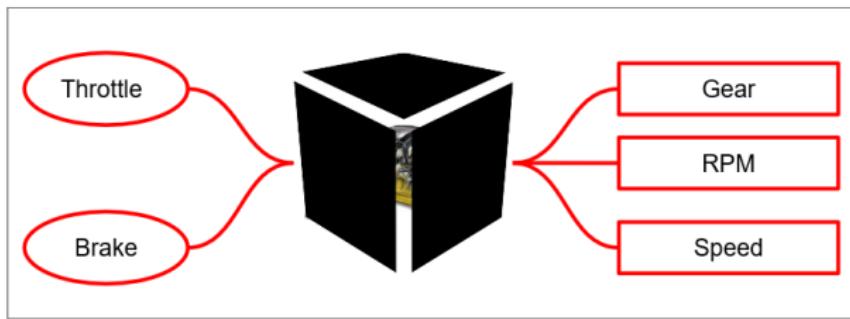
# Black Box Assumption



## Inputs & Outputs

The Inputs are Piece Wise Constant (PWC) Functions, the Outputs are PWC functions (Gear) or Continuous Functions.

# Black Box Assumption



## Black Box Assumption

- less information
- an more general approach (interesting by an industrial point of view)

# The requirements: Signal Temporal Logic (STL)

Signal temporal logic is:

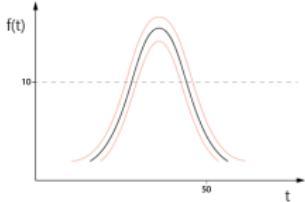
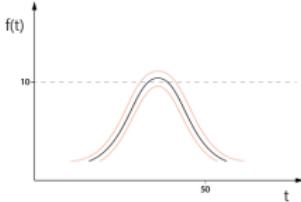
- a linear continuous time temporal logic.
- the atomic predicates are of the form  $\mu(\vec{X}) := [g(\vec{X}) \geq 0]$  where  $g : \mathbb{R}^n \rightarrow \mathbb{R}$  is a continuous function.
- the syntax is

$$\phi := \perp \mid \top \mid \mu \mid \neg\phi \mid \phi \vee \phi \mid \phi \mathbf{U}_{[T_1, T_2]} \phi, \quad (1)$$

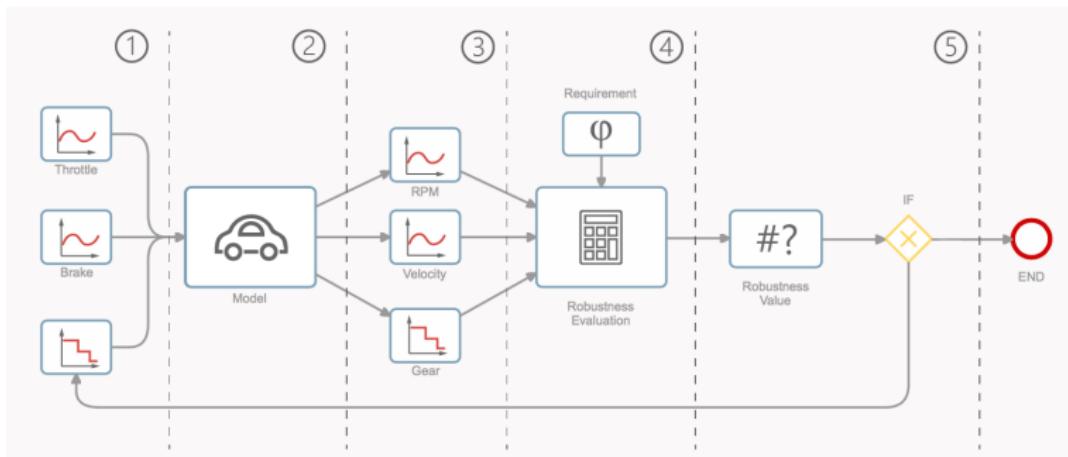
## Example

$$\phi_1 := F_{[0, 50]} |X_1 - X_2| > 10$$

- ① **The Booleans semantics:** if a given path satisfies or not a given STL formula.
- ② **The Quantitative semantics:** How much a given path satisfies or not a given STL formula.



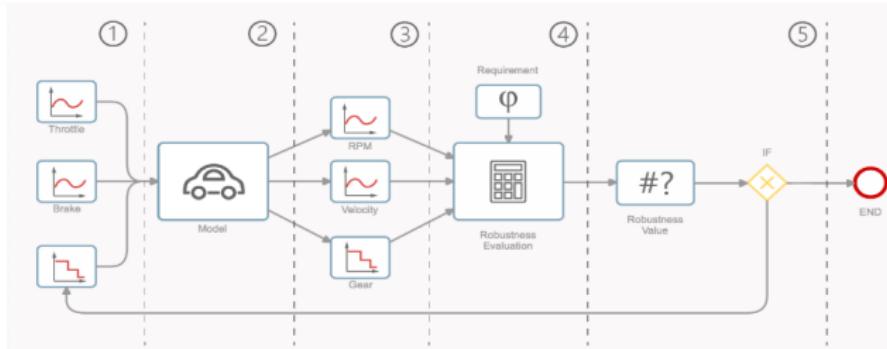
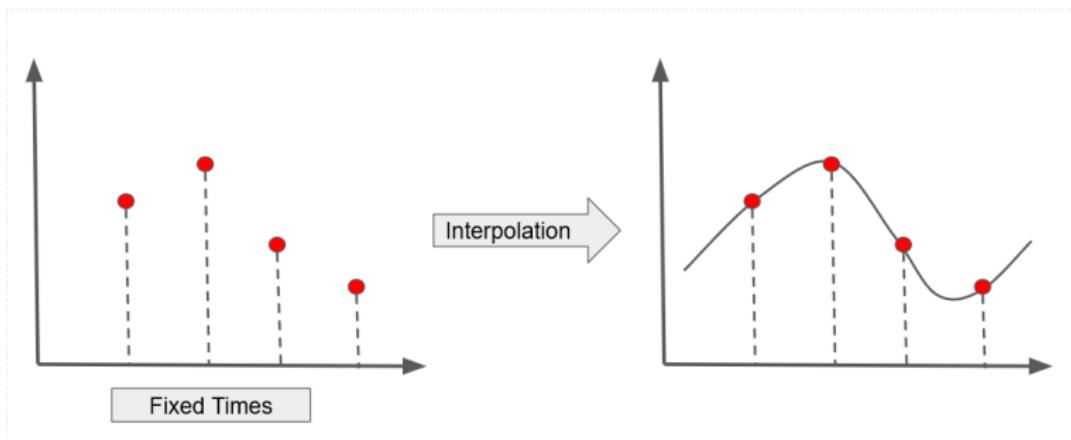
# Search-Based Testing



## Falsification

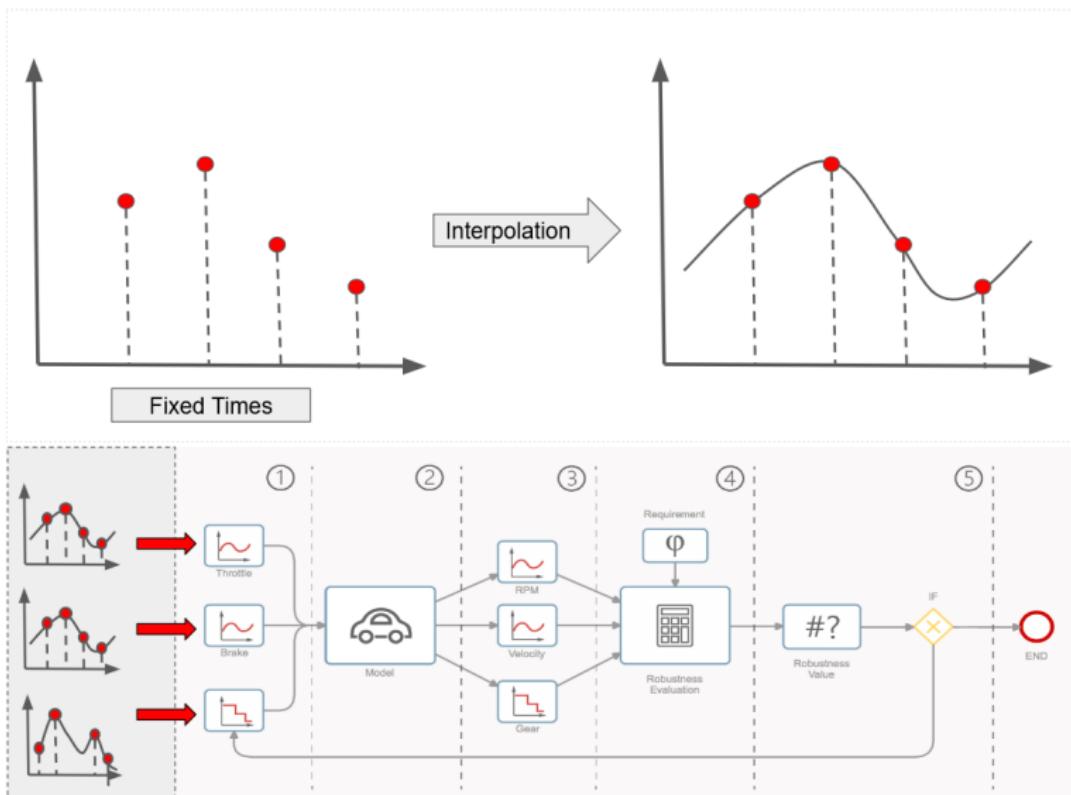
- Goal: Find the input functions (1) which violate the requirements (4)
- Problems
  - 1 Falsify with a low number of simulations  $\Rightarrow$  Active Learning
  - 2 Functional Input Space(!!)  $\Rightarrow$  Adaptive Space Parameterization

# Fixed Parameterization



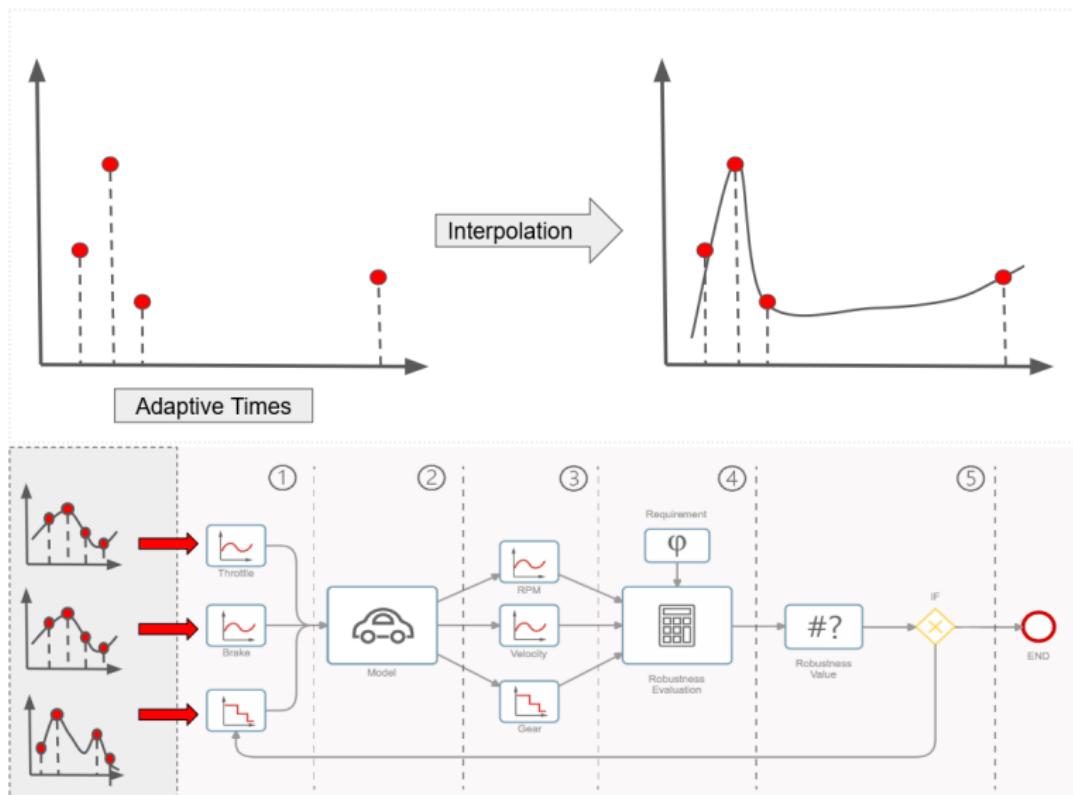
$n$  adaptive control points  $\Rightarrow n$  variable to optimize

# Fixed Parameterization



$n$  fixed control points  $\Rightarrow n$  variable to optimize

# Adaptive Parameterization



$n$  adaptive control points  $\Rightarrow$   $2n$  variable to optimize

# Domain Estimation Problem

## Domain Estimation Problem

Consider a function  $\rho : \Theta \rightarrow \mathbb{R}$  and an interval  $I \subseteq \mathbb{R}$ . We define the *domain estimation problem* as the task of identifying the set:

$$\mathcal{B} = \{\theta \in \Theta | \rho(\theta) \in I\} \subseteq \Theta \quad (2)$$

In practice, if  $\mathcal{B} \neq \emptyset$ , we will limit us to identify a subset  $B \subseteq \mathcal{B}$  of size  $n$ .

## Falsification $\sim$ Domain estimation problems

$$\mathcal{B} = \{\theta \in \Theta | \rho(\theta) \in (-\infty, 0)\} \subseteq \Theta$$



## Gaussian Processes

# Gaussian Processes

## Definition

A random variable  $f(\theta), \theta \in \Theta$  is a GP

$$f \sim \mathcal{GP}(m, k) \iff (f(\theta_1), f(\theta_2), \dots, f(\theta_n)) \sim \mathcal{N}(\mathbf{m}, K)$$

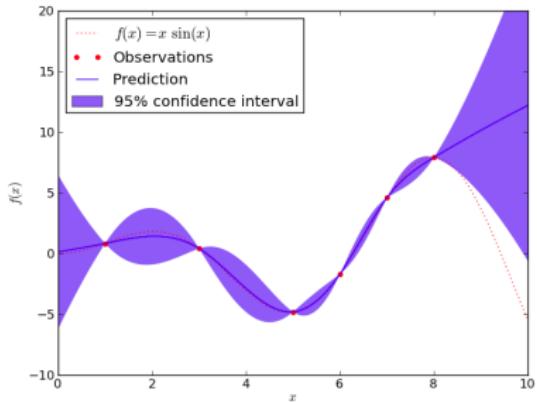
where  $\mathbf{m} = (m(\theta_1; h_1), m(\theta_2; h_1), \dots, m(\theta_n; h_1))$  and  $K_{ij} = k(f(\theta_i), f(\theta_j); h_2)$

## Prediction

$$\{f(\theta_1), \dots, f(\theta_n), f(\theta')\} \sim \mathcal{N}(\mathbf{m}', K')$$

$$\mathbb{E}(f(\theta')) = (k(\theta', \theta_1), \dots, k(\theta', \theta_N)) K_N^{-1} r$$

$$\text{var}(f(\theta')) = k(\theta', \theta') - K(\theta, r) K_N^{-1} K(\theta, r)^T$$



# Domain Estimation Problem

## Domain Estimation Problem

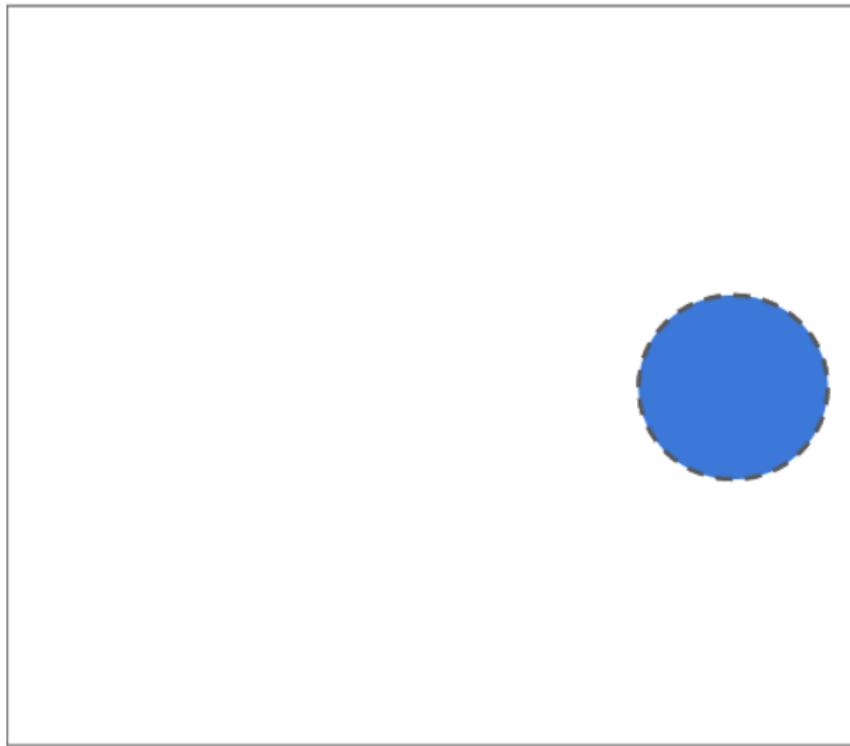
- Train Set:  $K(\rho) = \{(\theta_i, \rho(\theta_i))\}_{i \leq n}$  (the partial knowledge)
- Gaussian Process:  $\rho_K(\theta) \sim GP(m_K(\theta), \sigma_K(\theta))$  (the partial model)

$$P(\rho_K(\theta) < 0) = CDF\left(\frac{0 - m_K(\theta)}{\sigma_K(\theta)}\right)$$

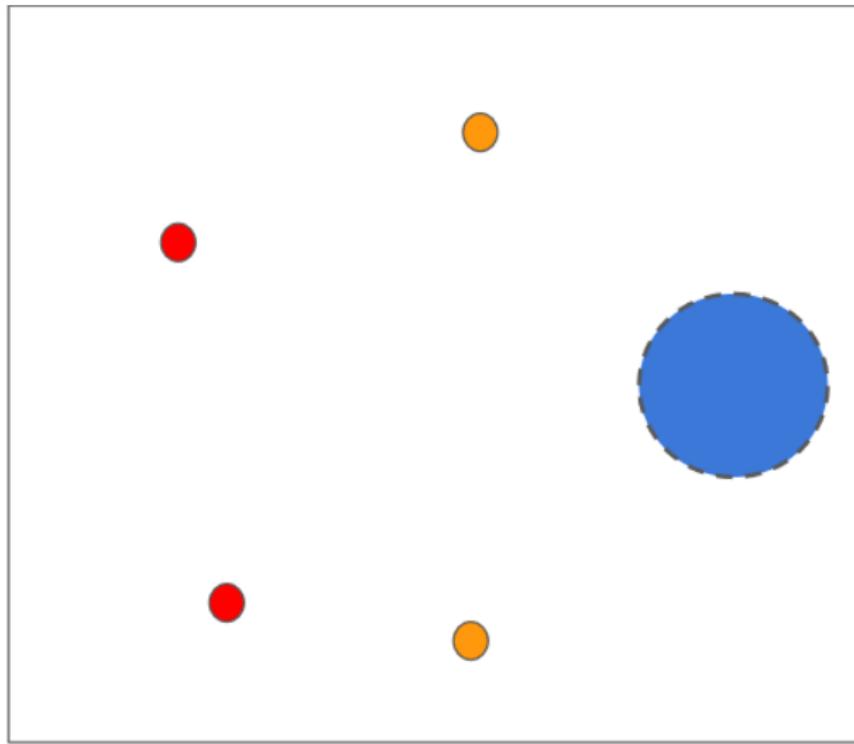
## Simple Idea

Iteratively explore the area which is more probable to falsify the system by sampling from  $P(\rho_K(\theta) < 0)$ .

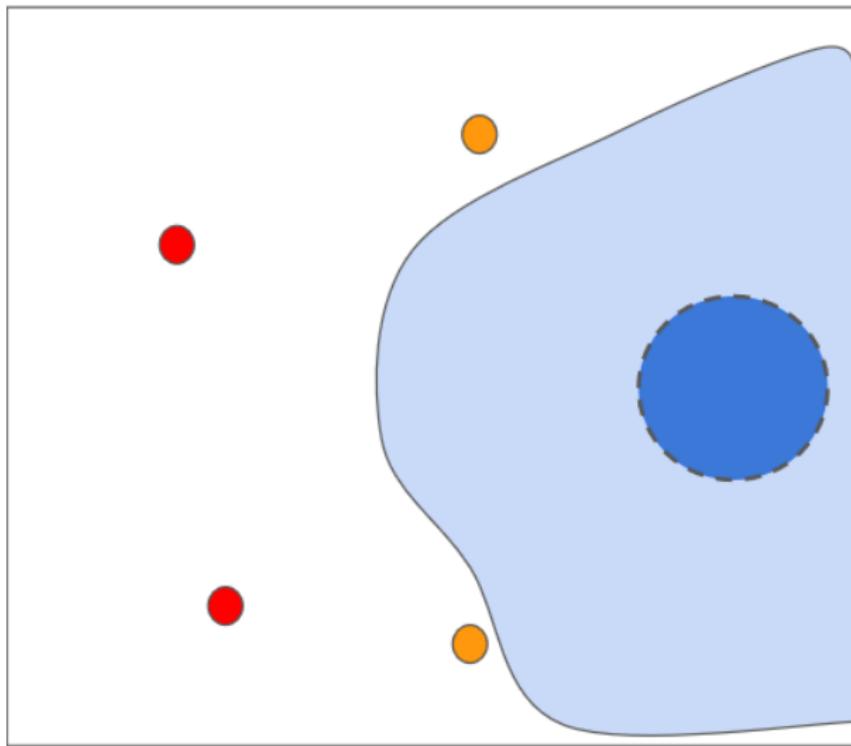
# Algorithm - I



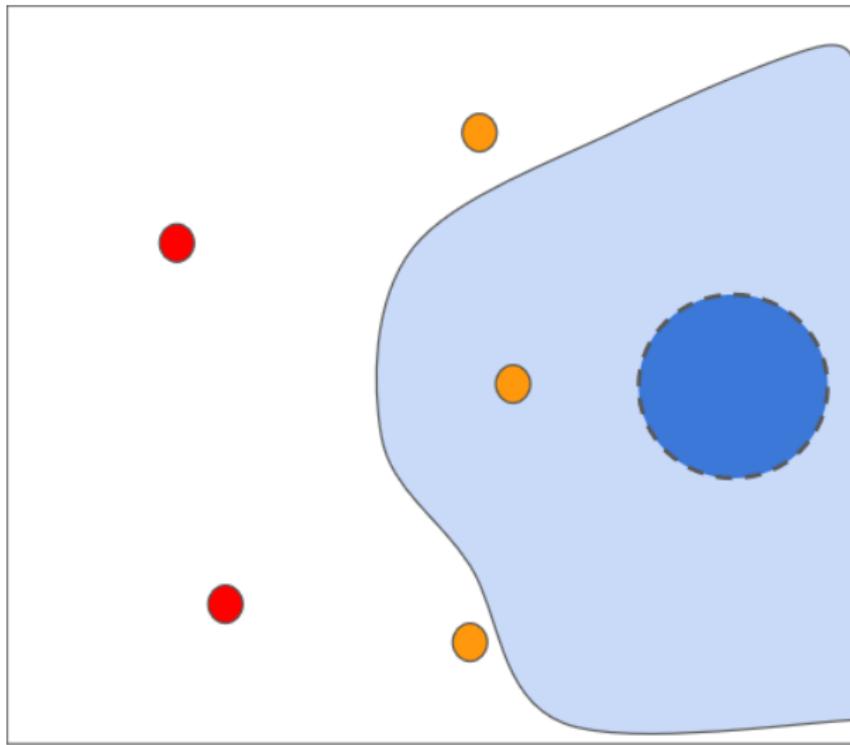
## Algorithm - II



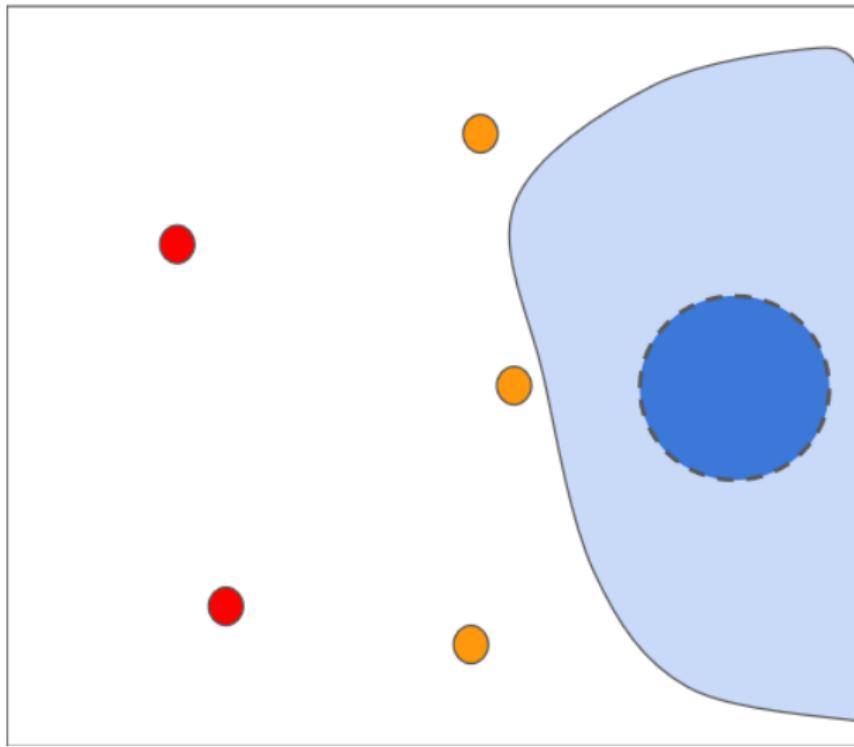
# Algorithm - III



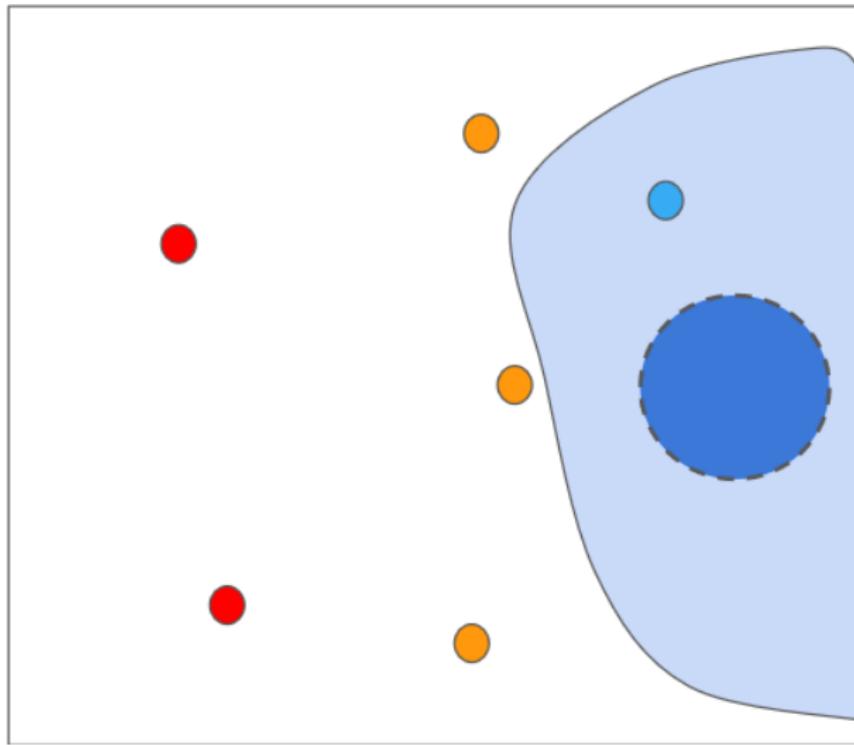
# Algorithm - IV



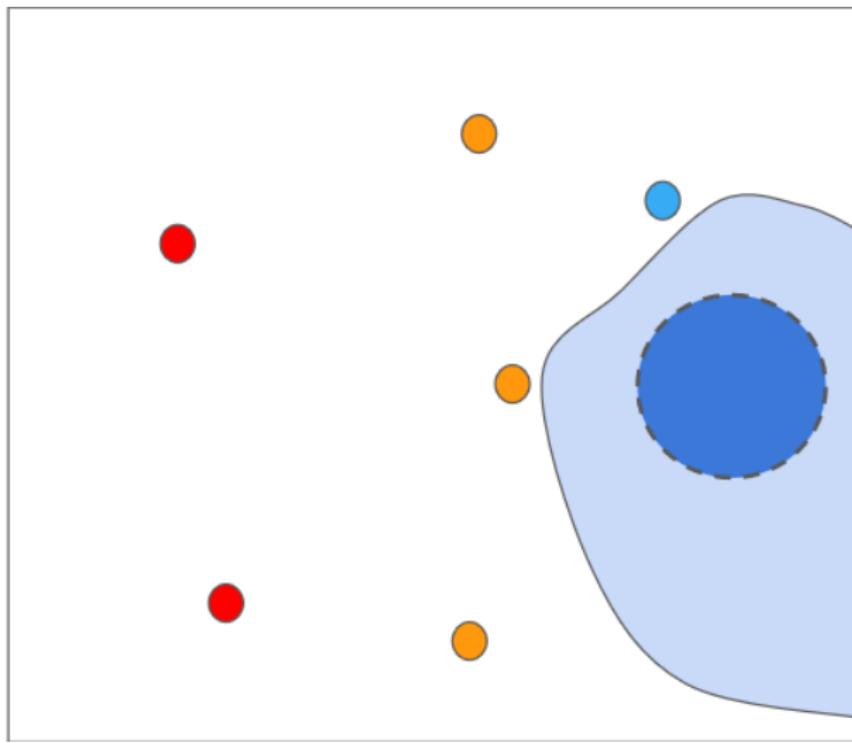
# Algorithm - V



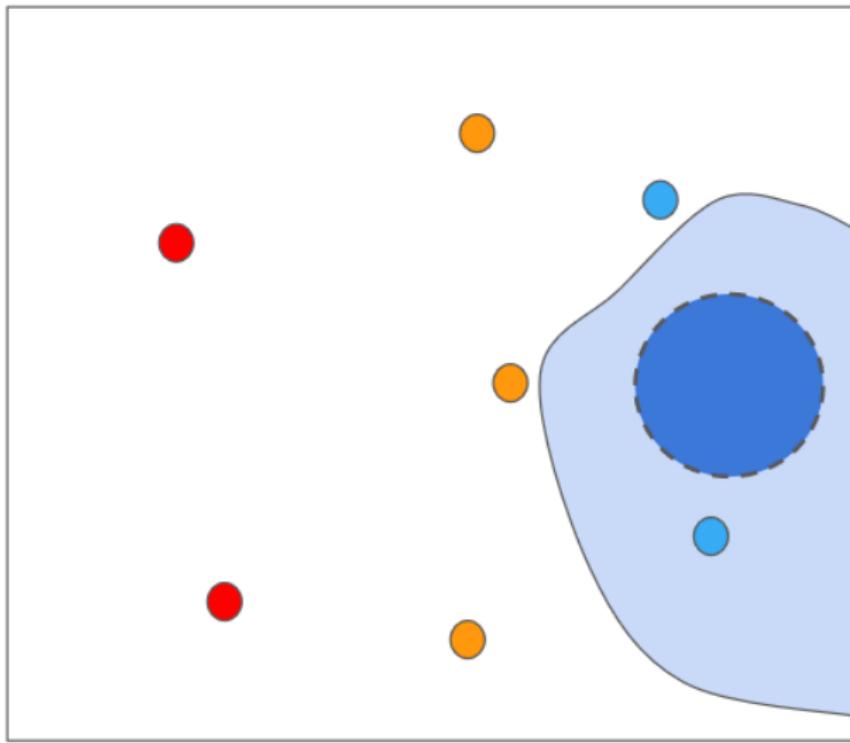
# Algorithm - VI



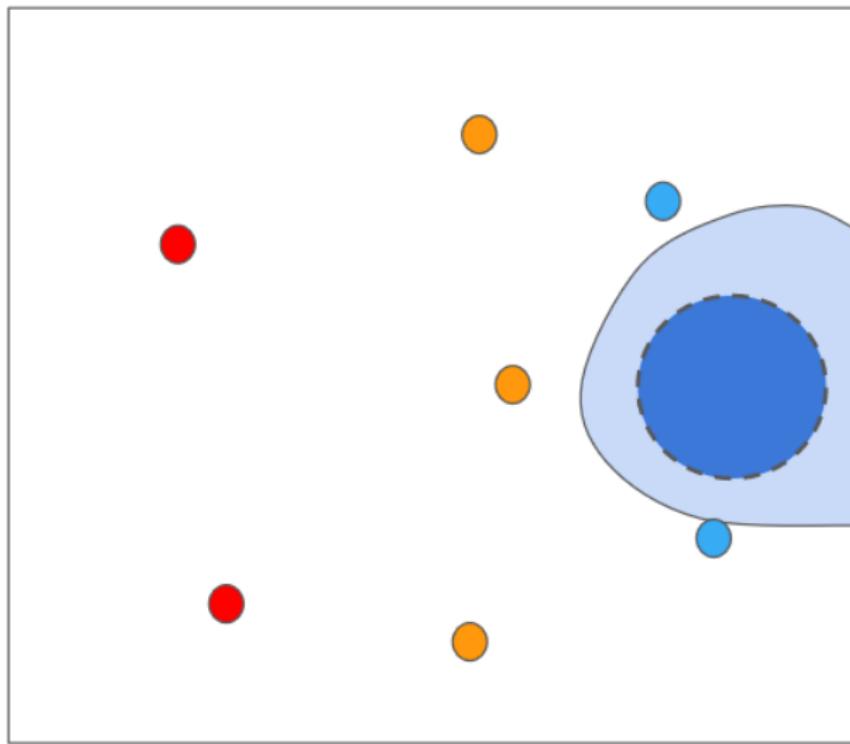
## Algorithm - VII



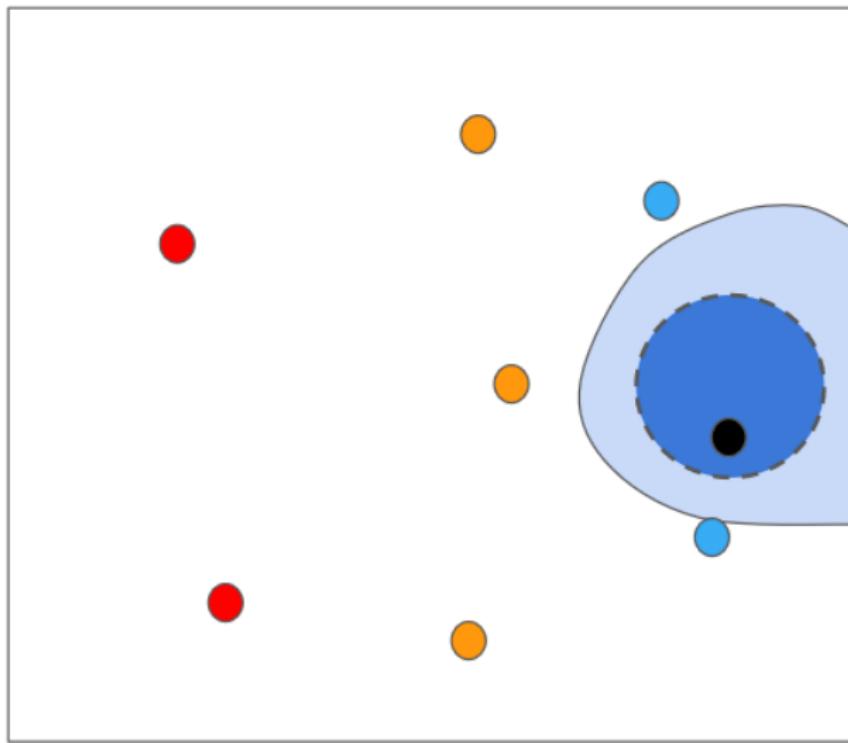
## Algorithm - VIII



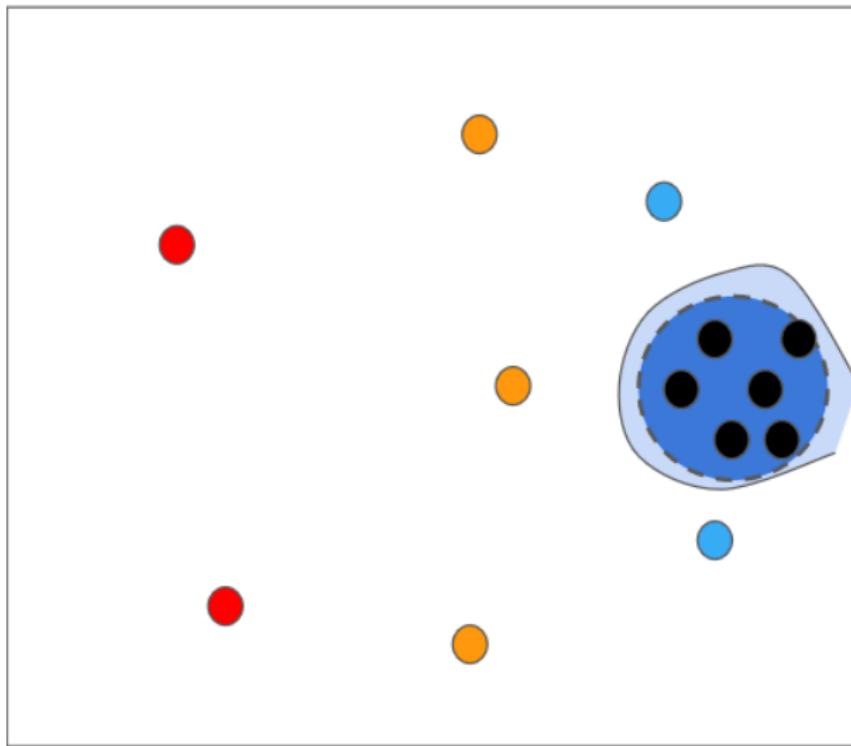
# Algorithm - IX



# Algorithm - X



# Algorithm - XI



# Probabilistic Approximation Semantics

## Definition ( $\mathcal{L}_0$ and $\mathcal{L}$ )

$\mathcal{L}_0$  : [ $\subset$  STL]: atomic propositions +  $\phi_1 \mathbf{U}_T \phi_2$ ,  $\mathbf{F}_T \phi$ ,  $\mathbf{G}_T \phi$ ,  
that cannot be equivalently written as Boolean combinations of simpler formulas;

$$\mathbf{F}_T(\phi_1 \vee \phi_2) \equiv \mathbf{F}_T\phi_1 \vee \mathbf{F}_T\phi_2 \notin \mathcal{L}_0$$

$\mathcal{L}$  : the boolean connective closure of  $\mathcal{L}_0$ .

## Definition (Probabilistic Approximation Semantics of $\mathcal{L}$ )

The probabilistic approximation function  $\gamma : \mathcal{L} \times Path^{\mathcal{M}} \times [0, \infty) \rightarrow [0, 1]$  is defined by:

- $\gamma(\phi, \theta, t) = P(f_{K(\phi)}(\theta) > 0)$
- $\gamma(\neg\psi, \theta, t) = 1 - \gamma(\psi, \theta, t)$
- $\gamma(\psi_1 \wedge \psi_2, \theta, t) = \gamma(\psi_1, \theta, t) * \gamma(\psi_2, \theta, t)$
- $\gamma(\psi_1 \vee \psi_2, \theta, t) = \gamma(\psi_1, \theta, t) + \gamma(\psi_2, \theta, t) - \gamma(\psi_1 \wedge \psi_2, \theta, t)$

# Test Case & Results

## Automotive Requirements

- $\phi_1(\bar{v}, \bar{\omega}) = \mathbf{G}_{[0,30]}(v \leq \bar{v} \wedge \omega \leq \bar{\omega})$  (in the next 30 seconds the engine and vehicle speed never reach  $\bar{\omega}$  rpm and  $\bar{v}$  km/h, respectively)
- $\phi_2(\bar{v}, \bar{\omega}) = \mathbf{G}_{[0,30]}(\omega \leq \bar{\omega}) \rightarrow \mathbf{G}_{[0,10]}(v \leq \bar{v})$  (if the engine speed is always less than  $\bar{\omega}$  rpm, then the vehicle speed can not exceed  $\bar{v}$  km/h in less than 10 sec)
- $\phi_3(\bar{v}, \bar{\omega}) = \mathbf{F}_{[0,10]}(v \geq \bar{v}) \rightarrow \mathbf{G}_{[0,30]}(\omega \leq \bar{\omega})$  (the vehicle speed is above  $\bar{v}$  km/h than from that point on the engine speed is always less than  $\bar{\omega}$  rpm)

Adaptive DEA			Adaptive GP-UCB			S-TaLiRo		
Req	nval	times	nval	times	nval	times	Alg	
$\phi_1$	<b><math>4.42 \pm 0.53</math></b>	$2.16 \pm 0.61$	<b><math>4.16 \pm 2.40</math></b>	$0.55 \pm 0.30$	$5.16 \pm 4.32$	$0.57 \pm 0.48$	UR	
$\phi_1$	<b><math>6.90 \pm 2.22</math></b>	$5.78 \pm 3.88$	$8.7 \pm 1.78$	$1.52 \pm 0.40$	$39.64 \pm 44.49$	$4.46 \pm 4.99$	SA	
$\phi_2$	<b><math>3.24 \pm 1.98</math></b>	$1.57 \pm 1.91$	$7.94 \pm 3.90$	$1.55 \pm 1.23$	$12.78 \pm 11.27$	$1.46 \pm 1.28$	CE	
$\phi_2$	<b><math>10.14 \pm 2.95</math></b>	$12.39 \pm 6.96$	$23.9 \pm 7.39$	$9.86 \pm 4.54$	$59 \pm 42$	$6.83 \pm 4.93$	SA	
$\phi_2$	<b><math>8.52 \pm 2.90</math></b>	$9.13 \pm 5.90$	$13.6 \pm 3.48$	$4.12 \pm 1.67$	$43.1 \pm 39.23$	$4.89 \pm 4.43$	SA	
$\phi_3$	<b><math>5.02 \pm 0.97</math></b>	$2.91 \pm 1.20$	$5.44 \pm 3.14$	$0.91 \pm 0.67$	$10.04 \pm 7.30$	$1.15 \pm 0.84$	CE	
$\phi_3$	<b><math>7.70 \pm 2.36</math></b>	$7.07 \pm 3.87$	$10.52 \pm 1.76$	$2.43 \pm 0.92$	$11 \pm 9.10$	$1.25 \pm 1.03$	UR	

# Conditional Safety Property

## Falsification of Conditional Safety Property

$$\mathbf{G}_T(\phi_{cond} \rightarrow \phi_{safe})$$

**Goal:** exploring cases in which the formula is falsified but the antecedent condition holds  
**Domain Estimation Approach:**

- sampling to achieve  $\phi_{cond}$
- sampling to falsify  $\phi_{safe}$

Adding one sampling routine in the Domain Estimation Algorithm.

## A formula which cannot be falsified!

$$\mathbf{G}_{[0,30]}(\omega \leq 3000 \rightarrow v \leq 100)$$

- GP-UCB: 43% of input satisfying  $\omega \leq 3000$
- DEA: 87% of input satisfying  $\omega \leq 3000$

# Challenges & Further studies

## Results

### Our Approach

- permits to reduce the minimum number of evaluations needed to falsify a model (respect to the state-of-art S-TaLiro Toolbox <sup>1</sup>)
- can be easily customize to solve Conditional Safety Property

## Further Studies

- Analyzing the sparse approximation techniques which reduces the computational cost of the Gaussian Processes
- Improving the sampling approach of Domain Estimation Algorithm (MCMC, etc..)

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<sup>1</sup> Annpureddy, Yashwanth, et al. "S-taliro: A tool for temporal logic falsification for hybrid systems". International Conference on Tools and Algorithms for the Construction and Analysis of Systems. Springer Berlin Heidelberg, 2011.

# Thank You